Minimal Hierarchical Collision Detection

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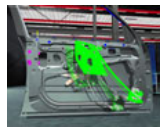
Motivation

- Fundamental operation:
  - Virtual prototyping
  - Rigid bodies
  - Interaction in VR
  - Haptic rendering



General requirements

- No assumptions about input
- No assumption about motion
- Complexity: ≫10,000 polygons / object
- Witness(es)
- Construction of aux data structures not too slow
- Small memory footprint

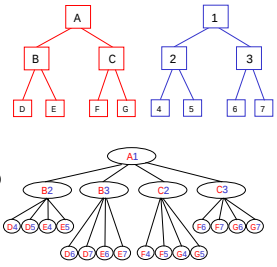
Related Work

- R*-trees [Beckmann, Kriegel, et al., 1990]
- Sphere trees [Hubbard, 1996]
- OBB trees [Gottschalk, et al., 1996]
- DOP trees [Zachmann, 1998; Klosowski, et al., 1998]
- BOXTREE [Barequet, at al., 1996]

Setting

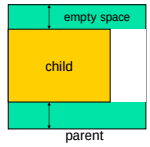
Hierarchical traversal scheme:

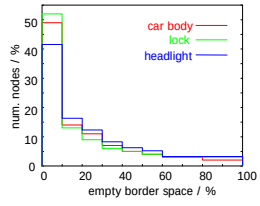
traverse(X, Y)
if X,Y do not overlap **then**
  return
if X,Y are leaves **then**
  check polygons
else
  for all children pairs **do**
    traverse(X_i, Y_j)



Restricted Boxtrees

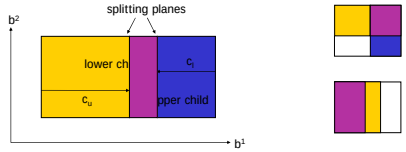
- Observation: child boxes fit fairly tightly into parent box on most sides





Definition

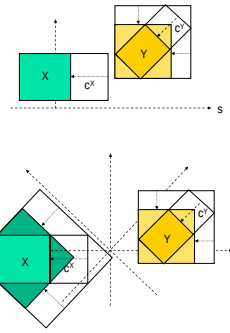
- Combination of k-d tree and AABB:



- Storage: 1 float, 1 axis ID, 1 pointer

Overlap Tests

1. Re-alignment:
   - 12 FLOPs
2. SAT:
   - 82 FLOPs
3. SAT lite:
   - 24 FLOPs
4. Sphere test:
   - 29 FLOPs



General Optimization

- Factorization of overlap test costs:
  c_1 = node-specific
  c_2 = pair-specific
- Brute-force:

$$C(X,Y) = 2c_1 + c_2 + 4(2c_1 + c_2) = 10c_1 + 5c_2$$

- Eager computation:

$$C(X,Y) = 0c_1 + c_2 + 2c_1 + 4c_2 \quad = 2c_1 + 5c_2$$

- E.g.: 1.5 mult + 2 add + 5 comp vs. 12 FLOPs

Constructing Restricted Boxtrees

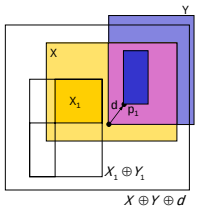
- Approach: top-down
  1. Compute BV covering input
  2. Split input into two subsets
- Splitting criterion:
  - Expected traversal cost:

$$C(X,Y) = 4 + \sum_{i,j=1,2} P(X_i,Y_j)C(X_i,Y_j)$$
$$\approx 4\left(1 + P(X_1,Y_1) + \cdots + P(X_2,Y_2)\right)$$

- Estimation of $P(X_i,Y_j)$:

$$X_i \cap Y_j \neq \varnothing \Leftrightarrow \mathbf{p}_i \in X_i \oplus Y_j$$

$$P(X_i,Y_j) = \frac{\mathrm{vol}(X_i \oplus Y_j)}{\mathrm{vol}(X \oplus Y)}$$
$$\approx \frac{\mathrm{vol}(X_i) + \mathrm{vol}(Y_j)}{\mathrm{vol}(X) + \mathrm{vol}(Y)}$$



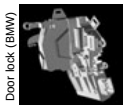
Algorithm

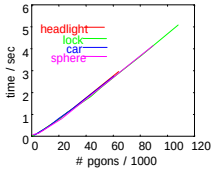
- Try three cases:
  1. Lower and upper child boxes
  2. Both lower/upper
  3. Perpendicular
- Find "good" splitting plane
- Find good "seeds"
- Split set of polygons
- Complexity:

$$T(n) = cn + T(\alpha n) + T((1-\alpha)n) \in O(n)$$

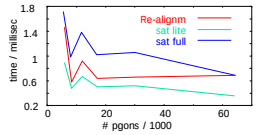
Results

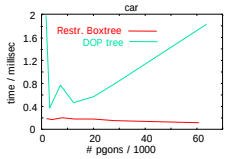
- Suite:
- Platform:
  - PentiumIII, 1GHz
- Construction:

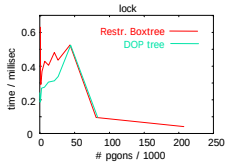




Collision detection time







Summary

- New hierarchical BV structure (*Restricted Boxtree*) with extremely small memory footprint (9 bytes/node)
- Very efficient overlap tests for restricted boxes (down to 8.5 FLOPs per BV pair)
- General optimization technique
- Theoretical argument for construction criterion
- Comparison with DOP trees

Future Work

- Other applications (ray tracing, occlusoin culling, ...)
- "Look-ahead" during construction