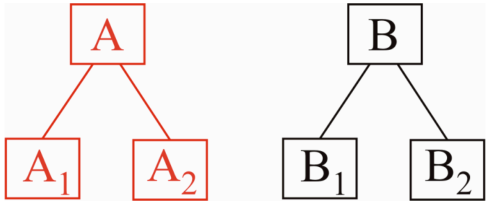
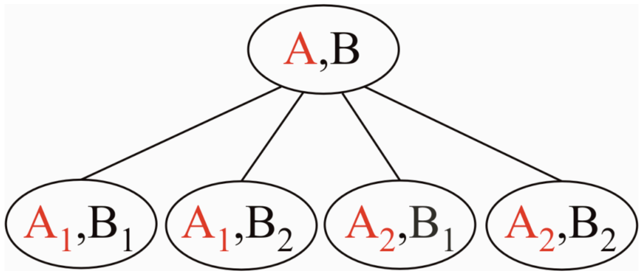
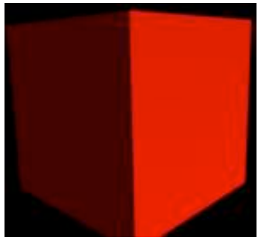
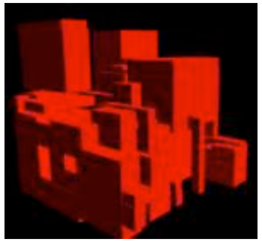
Expected Running Time of Hier. Coll. Det.

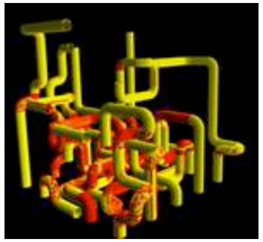
- Simultaneous traversal of **two** BVHs equals traversal of **one** *bounding volume test tree* (BVTT)











- The Cost formula:

$$T = N_v C_v + N_p C_p + N_u C_u + C_i$$

$$\begin{aligned} N_v, C_v &= \text{ num. and avg. costs of BV overlap tests, resp.} \\ N_p, C_p &= \text{ num. and avg. costs of primitive intersection tests} \\ N_u, C_u &= \text{ num. and avg. costs of BV updates, resp.} \\ C_i &= \text{ initialization costs} \end{aligned}$$

- Worst-case: $O(n^2)$
- Question: average case?
  - Clearly: $N_p \leq \frac{1}{2} N_v$ and $N_u \leq N_v$
- Task: determine average N_v = # nodes in the BVTT that are visited *on average*

Conditional Probability of Overlap

- Probability that child BVs overlap (on average):

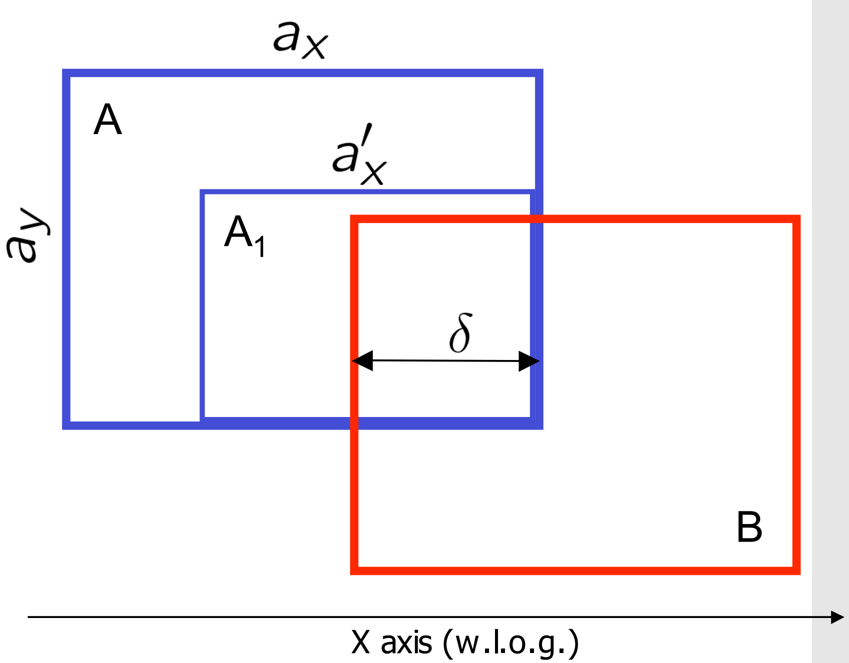
$$p_{ij}^{(l)} := Pr[A_i^{(l)} \cap B_j^{(l)} \neq \varnothing \mid A^{(l-1)} \cap B^{(l-1)} \neq \varnothing]$$

- Assumptions to simplify things:
  - Overlap δ of A and B along x axis is known
  - "BV diminishing factor" α_x:

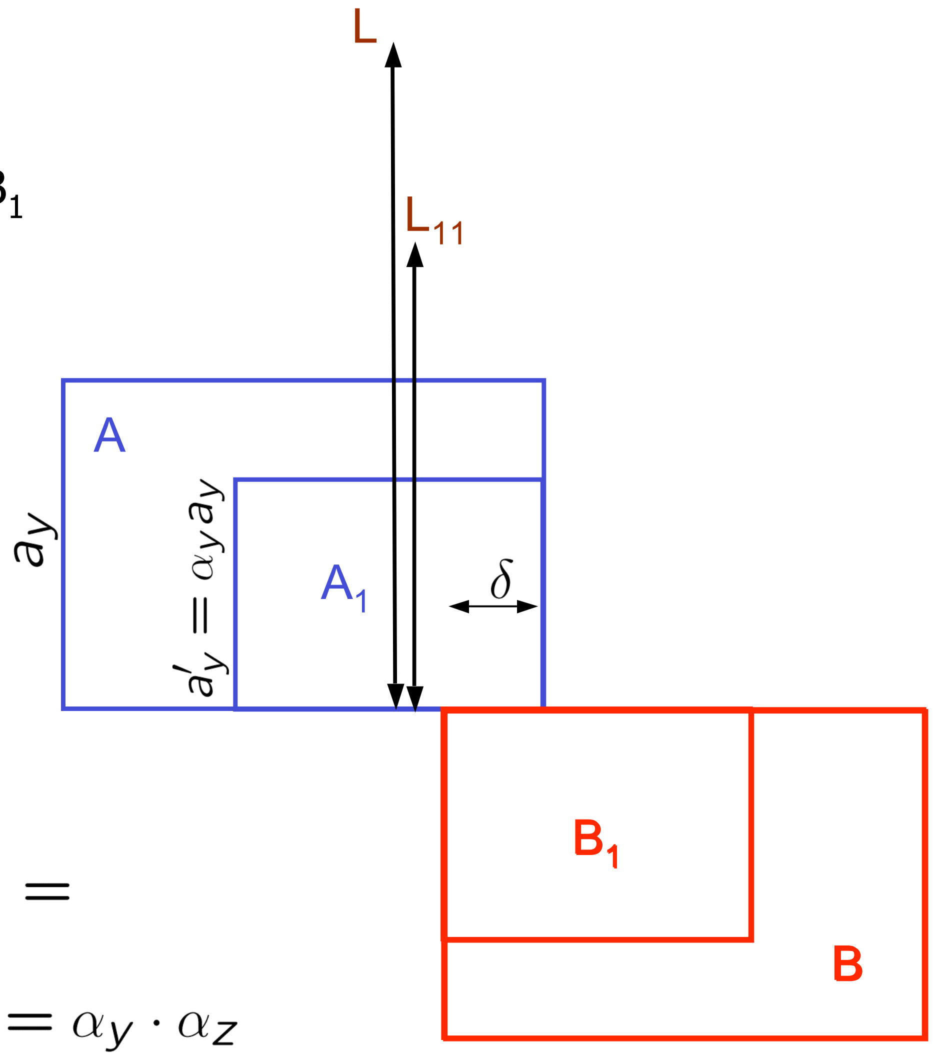
    $$a'_x = \alpha_x a_x,\ a'_y = \alpha_y a_y, \ldots$$

  - Boxes are of same order on same level, i.e.,

    $$b_x = a_x,\ b'_x = a'_x,\ \ldots$$



- First, compute p_{11}, i.e., probability of A_1 overlapping B_1 under condition that parent boxes overlap
- Argument analogous to Minkowski sums



- Probability is

$$Pr[A_1 \text{ overlaps } B_1 \ldots] =$$

$$= \frac{\text{area}(L)}{\text{area}(L_{11})} = \alpha_y \cdot \alpha_z$$

- Other child BV overlap probabilities
  - Similarly determine p_{21}, etc. …
  - Turns out that all p_{ij} are equal, i.e., $p_{21} = p_{12} = p_{22} = \alpha_y \alpha_z$

- A lot depends on how many BV pairs on level l overlap along x axis
  - Some child BVs x-overlap by same amount as parent BVs
  - Some child BVs don't x-overlap any more
  - Some child BVs x-overlap by a smaller amount than parents
  - Introduce distribution of x-overlaps (δ's)

level

$$\tilde{N}_v^{(l)}(\delta, \alpha_x)$$

Root BV overlap BV diminishing factor

- Overall expected number of nodes visited in BVTT:

$$\tilde{N}_V(n) = \sum_{l=1}^{\lg n} \tilde{N}_V^{(l)}(\delta, \alpha_X) \cdot \alpha_Y^l \cdot \alpha_Z^l$$

- Experimentally determined $\tilde{N}_V^{(l)}(\delta, \alpha_X)$

- Plots