Praktische Informatik 3, Uni Bremen

Unterlage zur VL vom 27.11.12

① Kontext $C_0 \equiv \{ (+) :: Int \to Int \to Int,\ length :: [\alpha] \to Int \}$

<u>ZZ</u>: $C_0 \vdash \lambda n\ xs.\ n + length\ xs :: Int \to [\alpha] \to Int$

Ableitungsbaum:

$$
\cfrac{
  \cfrac{
    \cfrac{}{C_1 \vdash (+) :: Int \to (Int \to Int)}\,\text{Var}
    \quad
    \cfrac{}{C_1 \vdash n :: Int}\,\text{Var}
  }{C_1 \vdash (+)\ n :: Int \to Int}
  \qquad
  \cfrac{
    \cfrac{}{C_1 \vdash length :: [\alpha] \to Int}\,\text{Var}
    \quad
    \cfrac{}{C_1 \vdash xs :: [\alpha]}\,\text{Var}
  }{C_1 \vdash length\ xs :: Int}\,\text{App}
}{
  \cfrac{C_1 \hat{=} C_0,\ n :: Int,\ xs :: [\alpha] \vdash n + length\ xs :: Int}{
    \cfrac{C_0,\ n :: Int \vdash \lambda xs.\ n + length\ xs :: [\alpha] \to Int}{C_0 \vdash \lambda n\ xs.\ n + length\ xs :: Int \to [\alpha] \to Int}\,\text{Abs}
  }\,\text{Abs}
}\,\text{App}
$$

Die baumartige Darstellung wird schnell unübersichtlich, daher bevorzugen wir im Folgenden die äquivalente <u>lineare Darstellung</u>:

		angewandte Regel	benutzte Voraussetzungen
1.	$C_1 \vdash (+) :: Int \to Int \to Int$	Var	
2.	$C_1 \vdash n :: Int$	Var	
3.	$C_1 \vdash (+)\ n :: Int \to Int$	App	[1,2]
4.	$C_1 \vdash length :: [\alpha] \to Int$	Var	
5.	$C_1 \vdash xs :: [\alpha]$	Var	
6.	$C_1 \vdash length\ xs :: Int$	App	[4,5]
7.	$C_1 \vdash (+)\ n\ (length\ xs) :: Int$	App	[3,6]
	$C_1 \equiv C_0,\ n :: Int,\ xs :: [\alpha]$		
8.	$C_0,\ n :: Int \vdash \lambda xs.\ n + length\ xs :: [\alpha] \to Int$	Abs	[7]
9.	$C_0 \vdash \lambda n\ xs.\ n + length\ xs :: Int \to [\alpha] \to Int$	Abs	[8]

(2) $\Gamma_0 = \{head :: [\alpha] \to \alpha,\ 3 :: Int,\ (+) :: Int \to Int \to Int\}$

1. $\Gamma_0, xs :: [\alpha] \vdash xs :: [\alpha]$ Var
2. $\Gamma_0, xs :: [\alpha] \vdash head :: [\alpha] \to \alpha$ Var
3. $\Gamma_0, xs :: [\alpha] \vdash head\ xs :: \alpha$ App [1,2]
4. $\Gamma_0, xs :: [\alpha] \vdash 3 :: Int$ Var
5. $\Gamma_0, xs :: [\alpha] \vdash (+) :: Int \to Int \to Int$ Var
6. $\Gamma_0, xs :: [\alpha] \vdash (+)\ 3 :: Int \to Int$ App [5,4]
7. $\Gamma_0, xs :: [\alpha] \vdash (+)\ 3\ (head\ xs)$ ↯

Hier müssen wir α in Zeile 3 mit Int instantiieren können, ohne dass der Typ von head zu $head :: [Int] \to Int$ wird.

↳ Aus diesem Grund führen wir den Allquantor ein, um Typvariablen zu binden, sowie die zwei Regeln Spec und Gen, um den Allquantor zu eliminieren bzw. einzuführen. Damit erhalten wir:

1. $\Gamma_0, xs :: [Int] \vdash xs :: [Int]$ Var
2. $\Gamma_0, xs :: [Int] \vdash head :: \forall\alpha.[\alpha] \to \alpha$ Var
3. $\Gamma_0, xs :: [Int] \vdash head :: [Int] \to Int$ Spec [2]
4. $\Gamma_0, xs :: [Int] \vdash head\ xs :: Int$ App [3,1]

⋮

7. $\Gamma_0, xs :: [Int] \vdash (+)\ 3 :: Int \to Int$ App [6,5]
8. $\Gamma_0, xs :: [Int] \vdash 3 + head\ xs :: Int$ App [7,4]
9. $\Gamma_0, \vdash \lambda xs.\ 3 + head\ xs :: [Int] \to Int$ Abs [8]

(3)

Sei $\Gamma_0 \triangleq \{ (:) :: \forall\alpha.\ \alpha \to [\alpha] \to [\alpha] \}$

Zz. $\Gamma_0 \vdash \mathit{head} = \lambda xs.\ \mathrm{case}\ xs\ \mathrm{of}\ y:ys \to y \ :: \forall\alpha.\ [\alpha] \to \alpha$

1. $\Gamma_0 \vdash (:) :: \forall\alpha.\ [\alpha] \to [\alpha] \to [\alpha]$ — Var
2. $\Gamma_0 \vdash (:) :: \beta \to [\beta] \to [\beta]$ — Spec [1], β frisch
3. $\Gamma_1 \vdash y :: \beta$ — Var
4. $\Gamma_1 \vdash (:)\ y :: [\beta] \to [\beta]$ — App [2,3]
5. $\Gamma_1 \vdash ys :: [\beta]$ — Var
6. $\Gamma_1 \vdash y:ys :: [\beta]$ — App [4,5]
7. $\Gamma_1 \vdash y :: \beta$ — Var
8. $\Gamma_0,\ xs :: [\beta] \vdash xs :: [\beta]$ — Var
9. $\Gamma_0,\ xs :: [\beta] \vdash \mathrm{case}\ xs\ \mathrm{of}\ y:ys \to y :: \beta$ — Cases [8,6,7]

   $\Gamma_1 \triangleq \Gamma_0,\ xs :: [\beta],\ y :: \beta,\ ys :: [\beta]$
10. $\Gamma_0 \vdash \lambda xs.\ \mathrm{case}\ xs\ \mathrm{of}\ y:ys \to y :: [\beta] \to \beta$ — Abs [9]
11. $\Gamma_0 \vdash \mathit{head} = \lambda xs.\ \mathrm{case}\ xs\ \mathrm{of}\ y:ys \to y :: \forall\beta.\ [\beta] \to \beta$ — Gen [10]

NB. $\forall\alpha.\ [\alpha] \to \alpha \equiv_\alpha \forall\beta.\ [\beta] \to \beta$

Der Name gebundener Variable ist irrelevant (α-Äquivalenz).

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Sei $\Gamma_0 \widehat{=} \{ tail :: \forall\alpha. [\alpha] \to [\alpha], head :: \forall\alpha. [\alpha] \to \alpha \}$

Wir haben:

$W(\Gamma_0, \lambda xs.\ tail\ (head\ xs))$

$= \underline{let}\ (\sigma_1, \tau_1) = W(\underbrace{\Gamma_0 \uplus \{xs :: \alpha\}}_{\Gamma_1}, tail\ (head\ xs))$ α frisch

$W(\Gamma_1, tail\ (head\ xs))$

$= \underline{let}\ (\sigma_2, \tau_2) = W(\Gamma_1, tail)$

$= (Id, [\beta] \to [\beta])$ β frisch

$(\sigma_3, \tau_3) = W(\Gamma_1, head\ xs)$

$W(\Gamma_1, head\ xs)$

$= let\ (\sigma_1, \tau_1) = \cancel{W}(~~head~~\ \Gamma_1, head)$

$= (Id, [\gamma] \to \gamma)$ γ frisch

$(\sigma_2, \tau_2) = W(\Gamma_1, xs)$

$= (Id, \alpha)$

$u_1 = unify([\gamma] \to \gamma, \alpha \to \delta)$ δ frisch

$= (\alpha \mapsto [\gamma], \delta \mapsto \gamma)$

$in\ (u_1, \gamma)$

$= ((\alpha \mapsto [\gamma], \delta \mapsto \gamma), \gamma)$

$u_2 = unify([\beta] \to [\beta], \gamma \to \varepsilon)$ ε frisch

$= (\gamma \mapsto [\beta], \varepsilon \mapsto [\beta])$

$\underline{in}\ (u_2 \circ u_1, u_2(\varepsilon))$

$= (\alpha \mapsto [[\beta]], \delta \mapsto [\beta], \varepsilon \mapsto [\beta], [\beta])$

$\underline{in}\ (\sigma_1, \sigma_1(\alpha \to \tau_1))$

$= (\sigma_1, [[\beta]] \to [\beta])$

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Damit ist

$$\sigma_1(\Gamma_0) \vdash \lambda xs.\ tail\ (head\ xs)) :: [[\beta]] \to [\beta]$$

$\shortparallel$

Γ_0 (da alle Typvariable in Γ_0 gebunden sind)

NB. Da wir $\lambda xs.\ tail\ (head\ xs)$ nicht an einen Bezeichner gebunden haben, ist diese Typvar. β nicht gebunden.

Dazu nötig wäre z.B.

let $f = \lambda xs.\ tail\ (head\ xs)$

in f

Dann ist $W(\Gamma_0, let \ldots) =$

$\underline{let}$ $(\sigma_4, \tau_4) = W(\Gamma_0, \lambda xs.\ tail\ (head\ xs))$

$= (\sigma_1, [[\beta]] \to [\beta])$

$(\sigma_5, \tau_5) = W(\sigma_1(\Gamma_0) \oplus \{f :: \forall \beta.\ [[\beta]] \to [\beta]\},$

$f)$

$= (id, \forall \beta.\ [[\beta]] \to [\beta])$

$\underline{in}$ $(\sigma_5 \cdot \sigma_4, \tau_5)$

$= (\Gamma_0, \forall \beta.\ [[\beta]] \to [\beta])$

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$W(\Gamma_0, \overset{xs.}{\text{tail}}\ (\text{head}\ xs))$

$\Gamma_0 = \{ \text{tail} :: \forall\alpha\,[\alpha] \to [\alpha],\ \text{head} :: \forall\alpha\,[\alpha] \to \alpha \}$

$\underline{\text{let}}\ (\sigma_1, \tau_1) = W(\Gamma_0 \cup \{xs : \alpha\}, \text{tail}\ (\text{head}\ xs))$ $\quad \alpha$ <u>frisch</u>

$\underline{\text{in}}\ (\sigma_1(\Gamma_0), \sigma_1(\alpha) \to \tau_1)$

~~$\{xs : xs\}$~~

$W(\Gamma_1, \text{tail}\ (\text{head}\ xs))$

$\underline{\text{let}}\ (\sigma_1, \tau_1) = \underline{W(\Gamma_1, \text{tail})}$

$= (id, [\beta] \to [\beta])$

$(\sigma_2, \tau_2) = W(\Gamma_1, \text{head}\ xs)$

$= \underline{\text{let}}\ (\sigma_1, \tau_1) = W(\Gamma_1, \text{head})$

$= (id, [\gamma] \to \gamma)$

$(\sigma_2, \tau_2) = W(\Gamma_1, xs)$

$= (id, \alpha)$

$\underline{\text{in}}\ u_1 = unify([\gamma] \to \gamma, \alpha \to \delta)$ $\quad \delta$ <u>frisch</u>

$= (\alpha \mapsto [\gamma], \delta \mapsto \gamma)$

$\underline{\text{in}}\ (u_1, \gamma)$

$u_2 = unify([\beta] \to [\beta], \gamma \to \varepsilon)$ $\quad \varepsilon$ frisch

$= (\gamma \mapsto [\beta], \varepsilon \mapsto [\beta])$

$\underline{\text{in}}\ (u_2 \cdot u_1, u_2(\varepsilon))$

$= (\gamma \mapsto [\beta], \alpha \mapsto [[\beta]], \delta \mapsto [\beta], \varepsilon \mapsto [\beta]),$
$[\beta])$

$\underline{\text{in}}\ (\Gamma_0^{\sigma_1}, \underline{[[\beta]] \to [\beta]})$

yields:

$p_1 \quad p_2 \quad i \qquad$ result type

$\gamma \to [(\delta, \varepsilon)]$

$\varepsilon \to [(\zeta, \eta)]$

γ

$\to [((\delta,\zeta), \eta)]$

given signature

$(\langle * \rangle) :: \text{Parse } a\ b \to \text{Parse } a\ c \to \text{Parse } a\ (b, c)$

$[a] \to (b, [a]) \qquad [a] \to (c, [a]) \qquad [a] \to [((b,c), [a])]$

matches with

$\gamma \to [a]$

$\delta \to b$

$\varepsilon \to [a]$

$\zeta \to c$

$\eta \to [a]$