Formal methods of software engineering

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Exercise Sheet 1

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Exercise 1.1 (Propositional Logic I)

(a) Use the equivalence rules introduced in the lecture to push all occurrences of the negation symbol "¬" next to the atoms in the following expressions:

(i) $\neg((A \to B) \vee ((A \to C) \wedge \neg A))$
(ii) $\neg(A \wedge \neg B) \to A$

(b) Below are two arguments in English. Translate each argument into logic using an appropriate dictionary, and check whether the argument is logically valid.

(i) If the king is in the room, then the courtiers laugh only if he laughs. The courtiers always laugh when the jester is in the room. The king never laughs when the jester is in the room. *Therefore*, either the king or the jester is not in the room.

(ii) If Jones did not meet Smith last night, then either Smith was a murderer, or Jones is telling a lie. If Smith was not a murderer, then Jones did not meet Smith last night, and the murder happened after midnight. If the murder happened after midnight, then either Smith was a murderer, or Jones is telling a lie, but not both. *Therefore*, Smith was a murderer.

Exercise 1.2 (Propositional Logic II)

(a) (i) Find formulae A, B, and C such that $\{A, B\}$, $\{A, C\}$, and $\{B, C\}$ are consistent, while $\{A, B, C\}$ is not.

(ii) For any n, find an inconsistent set of n formulae, of which every $n-1$ formulae are consistent.

(b) (i) Find four pairwise inconsistent non-contradictory formulae.

(ii) State the maximal number of pairwise inconsistent non-contradictory formulae with two atomic propositions p and q.

(c) Check the validity of the following rules:

$$\frac{A \wedge B}{A}\ (\wedge \text{ elimination}) \qquad \frac{A \quad B}{A \wedge B}\ (\wedge \text{ introduction})$$

$$\frac{\begin{array}{ccc}[A] & [B] & \\ \vdots & \vdots & \\ C & C & A \vee B\end{array}}{C}\ (\vee \text{ elimination}) \qquad \frac{A \quad \neg A}{B}\ (\neg \text{ elimination})$$

Exercise 1.3 (Propositional Logic III)

Suppose the engine of a car does not perform properly. We want to decide whether we should replace the engine, repair the engine, or replace auxiliary equipment. For the diagnosis, the following symptoms, intermediate conclusions and final decisions or diagnoses should be considered.

Variable	Meaning
black_exhaust	Engine fumes are black
blue_exhaust	Engine fumes are blue
low_power	Engine has low power
overheat	Engine overheats
ping	Engine emits a pinging sound under load
incorrect_timing	Ignition timing is incorrect
low_compression	Compression of engine is low
carbon_deposits	Cylinders have carbon deposits
clogged_filter	Air filter is clogged
clogged_radiator	Radiator is clogged
defective_carburetor	Carburetor is defective
worn_rings	Piston rings are worn
worn_seals	Valve seals are worn
replace_auxiliary	Replace auxiliary equipment
repair_engine	Repair engine
replace_engine	Replace engine

The following facts relate symptoms to intermediate conclusions (facts (i) through (vi)) and intermediate conclusions to final decisions (facts (vii) through (ix)).

(i) If the engine overheats and the ignition is correct, then the radiator is clogged.

(ii) If the engine emits a pinging sound under load and the ignition timing is correct, then the cylinders have carbon deposits.

(iii) If power output is low and the ignition timing is correct, then the piston rings are worn, or the carburetor is defective, or the air filter is clogged.

(iv) If the exhaust fumes are black, then the carburetor is defective, or the air filter is clogged.

(v) If the exhaust fumes are blue, then the piston rings are worn, or the valve seals are worn.

(vi) The compression is low if and only if the piston rings are worn.

(vii) If the piston rings are worn, then the engine should be replaced.

(viii) If carbon deposits are present in the cylinders or the carburetor is defective or valve seals are worn, then the engine should be repaired.

(ix) If the air filter or radiator is clogged, then that auxiliary equipment should be replaced.

Suppose the car owner complains that the engine overheats. Due to a recent engine check, it is known that the ignition timing is correct. What should be done to eliminate the problem?

Answer this question by translating the given information into a propositional CASL specification and checking with HETS which of the final decisions (diagnoses) follow from the symptoms.

Exercise 1.4 (Logical consequence or not?)

Evaluate the validity of the following argument. If it is a logical consequence, use the programs *SPASS*, *Fitch* and *Jitpro* to construct formal (resolution, natural deduction, tableau) proofs to show this. Otherwise, use *Tarski's World* to construct a counterexample.[1]

1	$\mathsf{Cube(a)} \vee (\mathsf{Cube(b)} \rightarrow \mathsf{Tet(c)})$
2	$\mathsf{Tet(c)} \rightarrow \mathsf{Small(c)}$
3	$(\mathsf{Cube(b)} \rightarrow \mathsf{Small(c)}) \rightarrow \mathsf{Small(b)}$
4	$\neg\mathsf{Cube(a)} \rightarrow \mathsf{Small(b)}$

Exercise 1.5 (Inconsistency)

Consider the set $\mathcal{T} = \{(A \wedge B) \rightarrow \neg A, C \vee A, \neg A \rightarrow A, B\}$. Use *SPASS*, *Fitch* and *Jitpro* to construct formal proofs showing that $\mathcal{T} \vdash \bot$.

Exercise 1.6 (New connectives)

Consider the following truth table for the ternary connective $\diamondsuit$.

P	Q	R	$\diamondsuit(P,Q,R)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Express $\diamondsuit$ using only the connectives $\vee$, $\wedge$, and $\neg$. Can you simplify the result such that the simplified sentence has no more than two occurrences each of P, Q, and R, and no more than six occurrences of the Boolean connectives $\vee$, $\wedge$, and $\neg$?

[1] SPASS is available within Hets, see `http://www.dfki.de/sks/hets`. Fitch and Tarski's World can be downloaded from an internal web page shown in the lecture. Jitpro is available under `http://ps.uni-sb.de/jitpro/prover.php`.